Willard Evan Bleick

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A note in this Journal, Ref. 1, discussed the problem of scheduling the direction p of constant momentum thrust of a rocket, which loses mass at a constant rate, so that it transfers to a known earth satellite orbit in minimum time T after launching. A numerical solution was obtained, using rectangular coordinates, for the case of fixed launching conditions. The method of Ref. 1 is extended here to solve the problem of orbital transfer of such a rocket with minimum fuel consumption. All of the symbols, units, and end conditions of Ref. 1 are used here without redefinition.

Statement of the Problem

The time of flight T in minimum fuel transfer must be longer than in the minimum time transfer of Ref. 1, unless these two trajectories turn out to be identical. This implies at least one interruption in rocket thrust during minimum fuel transfer. The problem solved here assumes exactly one such interruption, i.e. launch at t=0, thrust interruption at $t=t_1$, thrust resumption at $t=t_2$, and final thrust termination at transfer t=T. The problem of minimum fuel transfer is equivalent to the Lagrange calculus of variations problem of requiring the integral

$$J = \int_0^T (f+\lambda\varphi_1+\mu\varphi_2+\pi\varphi_3+\rho\varphi_4)dt \tag{1}$$

to be stationary, where f is the fuel consumption rate, λ,μ,π,ρ are continua of Lagrangian multipliers, and $\varphi_1=\varphi_2=\varphi_3=\varphi_4=0$ are the first order equations of rocket motion of Ref. 1. The f function and the

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rocket thrust per unit remaining mass function a are defined as follows: For $0<t<t_1$, $f=1$ and $a=c\dot{M}/(1-\dot{M}t)g$. For $t_1<t<t_2$, $f=0$ and $a=0$. For $t_2<t<T$, $f=1$ and $a=c\dot{M}/[1-\dot{M}(t+t_1-t_2)]g$. Note that for $t_2<t<T$ $\partial a/\partial t_1 = -\partial a/\partial t_2 = ga^2/c$. The varied time subinterval end points in Eq.(1) are taken as $t_1+\Delta t_1$, $t_2+\Delta t_2$ and $T+\Delta T$. The vanishing first variation δJ and its partial integration are computed as in Ref. 1. The coefficients of $\delta u, \delta v, \delta x, \delta y, \delta p$ in $\delta J=0$ give the Euler Eqs.(2) and (3), consisting of the adjoint equations

$$\begin{aligned} &\dot{\lambda}+\pi=0\ , &&\dot{\mu}+\rho=0\ , \\ &\dot{\pi}+g_{1x}\lambda+g_{2x}\mu=0\ , &&\dot{\rho}+g_{1y}\lambda+g_{2y}\mu=0\ , \end{aligned} \tag{2}$$

and the control equation

$$\tan p = \mu/\lambda\ . \tag{3}$$

The coefficient of ΔT in $\delta J=0$ gives, with the aid of Eq.(3), the transversality condition

$$(\mathbf{a}\cdot\Lambda)_T = (a\Lambda)_T = 1 \tag{4}$$

where the adjoint vector $\Lambda = i\lambda+j\mu$, $\Lambda = |\Lambda| = (\lambda^2+\mu^2)^{1/2}$, and $\mathbf{a} = a(i\cos p + j\sin p)$. The coefficients of Δt_1 and Δt_2 in $\delta J=0$ give, with the aid of Eq.(4), the corner conditions

$$H(t_1) = [a\Lambda]_{t_1}^{T} - \frac{g}{c}\int_{t_1}^{T} a^2\Lambda dt = 0, \quad H(t_2) = 0. \tag{5}$$

Eqs.(5) are equivalent, by the definition of a, to

$$\Lambda(t_1) = \Lambda(t_2) \tag{6}$$

and, by partial integration, to

$$\int_{t_2}^{T} a\dot{\Lambda}dt = 0\ . \tag{7}$$

<u>Numerical Solution</u>

Let $\lambda_i, \mu_i, \pi_i, \rho_i$, $i=1,2,3,4$, be four linearly independent solutions of the adjoint Eqs.(2) corresponding to the columns of the matrix E(t) of Ref. 1. The control angle p of Eq.(3) is defined by

$$\tan p = (\mu_1+l\mu_2+m\mu_3+n\mu_4)/(\lambda_1+l\lambda_2+m\lambda_3+n\lambda_4) \tag{8}$$

and its variation δp is obtained in terms of $\delta l, \delta m, \delta n$ by total differentiation as in Ref. 1. The Bliss fundamental formulas are obtained by assuming that a solution of the rocket motion equations $\varphi_1=\varphi_2=\varphi_3=\varphi_4=0$ has been found, corresponding to Eq.(8), which does not necessarily satisfy the terminal conditions at $t=T$ or the corner condition Eqs.(5). Using this solution and holding T fixed, but allowing t_1 and t_2 to vary, find the variation of the vanishing matrix integral

$$\int_0^T [\varphi_1, \varphi_2, \varphi_3, \varphi_4] E(t)dt = 0 \tag{9}$$

with the terminal constraints at $t=T$ removed. Since the columns of $E(t)$ satisfy the adjoint Eqs.(2), one obtains the system of Bliss formulas in the 1×4 matrix equation

$$\begin{aligned}[\delta u, \delta v, \delta x, \delta y]_T E(T) &+ [G(t_1)-(apF)_T]\Delta t_1 \\ &- [G(t_2)-(apF)_T]\Delta t_2 = [0, \delta l, \delta m, \delta n]A \end{aligned} \tag{10}$$

where the matrix A has been defined in Ref. 1, and where the matrix

$$G(t) = (apF)_t^T - \frac{g}{c}\int_t^T a^2 pF dt \tag{11}$$

where the 2×4 matrix $F(t)$ is the first two rows of $E(t)$, and where the matrix $p=[\cos p, \sin p]$. Substitution of

$$[\delta u, \delta v, \delta x, \delta y]_T = [U-u, V-v, X-x, Y-y]_T + [\dot{U}-\dot{u}, \dot{V}-\dot{v}, \dot{X}-\dot{x}, \dot{Y}-\dot{y}]_T \Delta T \tag{12}$$

into Eq.(10) gives four of the required six Newton-Raphson equations for the determination of $\Delta T, \Delta t_1, \Delta t_2, \delta l, \delta m, \delta n$ on the varied trajectory. The remaining two equations attempt to satisfy the corner condition Eqs.(5) on the varied trajectory. Involved here are the differentials

$$\begin{aligned} da &= \delta a + \dot{a}dt \\ &= (\partial a/\partial t_1)\Delta t_1 + (\partial a/\partial t_2)\Delta t_2 + (ga^2/c)dt \end{aligned} \tag{13}$$

and

$$\begin{aligned} d\Lambda &= \delta\Lambda + \dot{\Lambda}dt \\ &= [0, \delta l, \delta m, \delta n]F'p' + (\dot{\lambda}\cos p + \dot{\mu}\sin p)dt \end{aligned} \tag{14}$$

where the primes on F and p indicate matrix transposition. Use of Eqs.(6) and (14) yields the Newton-Raphson equation

$$\dot{\Lambda}(t_1)\Delta t_1 - \dot{\Lambda}(t_2)\Delta t_2 - [0, \delta l, \delta m, \delta n][F'p']_{t_1}^{t_2} \doteq \Lambda(t_2) - \Lambda(t_1) \ . \tag{15}$$

Use of Eqs.(13) and (14), and the first of Eqs.(5), yields the Newton-Raphson equation

$$(a\dot{\Lambda})_T\Delta T + (K-a\dot{\Lambda})_{t_1}\Delta t_1 - K(t_2)\Delta t_2 + [0,\delta l,\delta m,\delta n]G'(t_1) = -H(t_1) \tag{16}$$

where

$$K(t) = \frac{g}{c}[a^2\Lambda]_t^T - 2\left(\frac{g}{c}\right)^2\int_t^T a^3\Lambda dt \ . \tag{17}$$

The iteration to successive varied trajectories, using Eqs.(10), (12), (15) and (16), may be carried out as in Ref. 1. Two devices were used to stabilize the course of the iteration. The first was to adjust the m and n values of the new T,t_1,t_2,l,m,n sextuple, found by solving the Newton-Raphson equations, to satisfy the corner condition Eqs.(5) before proceeding with the next iteration. The second device was to modify the $[\dot{U},\dot{V},U,V,X,Y]_T$ terms in the Newton-Raphson equations, before solving these equations, so as to minimize the sum of the squares of the elements of $[U-u,V-v,X-x,Y-y]_T$.

The numerical example of minimum fuel transfer given here involves the same launching conditions, mass loss parameters and circular orbit used in the minimum time transfer of Ref. 1. The results for minimum fuel transfer are $T=0.353977$, $t_1=0.210293$, $t_2=0.275349$, $l=-0.820196$, $m=-0.708727$, $n=-1.181390$, and the transfer sector angle $B=0.189345$ rad. Since the minimum time trajectory of Ref. 1 gave $T=0.289869$, the net fuel saving in minimum fuel transfer over minimum time transfer is measured by $0.289869-0.353977+t_2-t_1 = 0.000948$, or an unspectacular one-third per cent. Figure 1 shows the trajectories and thrust directions for minimum time and minimum fuel transfer.

The semilogarithmic plots of Fig. 2 show the different behavior of Λ versus time in the two problems. For some reason there is much more difference than we expect. The curve increases monotonically for minimum time transfer. The curve for minimum fuel shows a rather char-

acteristic shape. It is large initially and decreasing; then it increases, and then decreases. If the final decreasing interval does not occur, larger values of T lead to lower values of fuel consumption, as may be partially inferred from Eq. (7).

Reference

[1] Bleick, W. E., "Orbital transfer in minimum time," AIAA Journal 1, 1229-1231 (1963).

Figures

Fig. 1 Trajectories and thrust directions

Fig. 2 Λ versus time

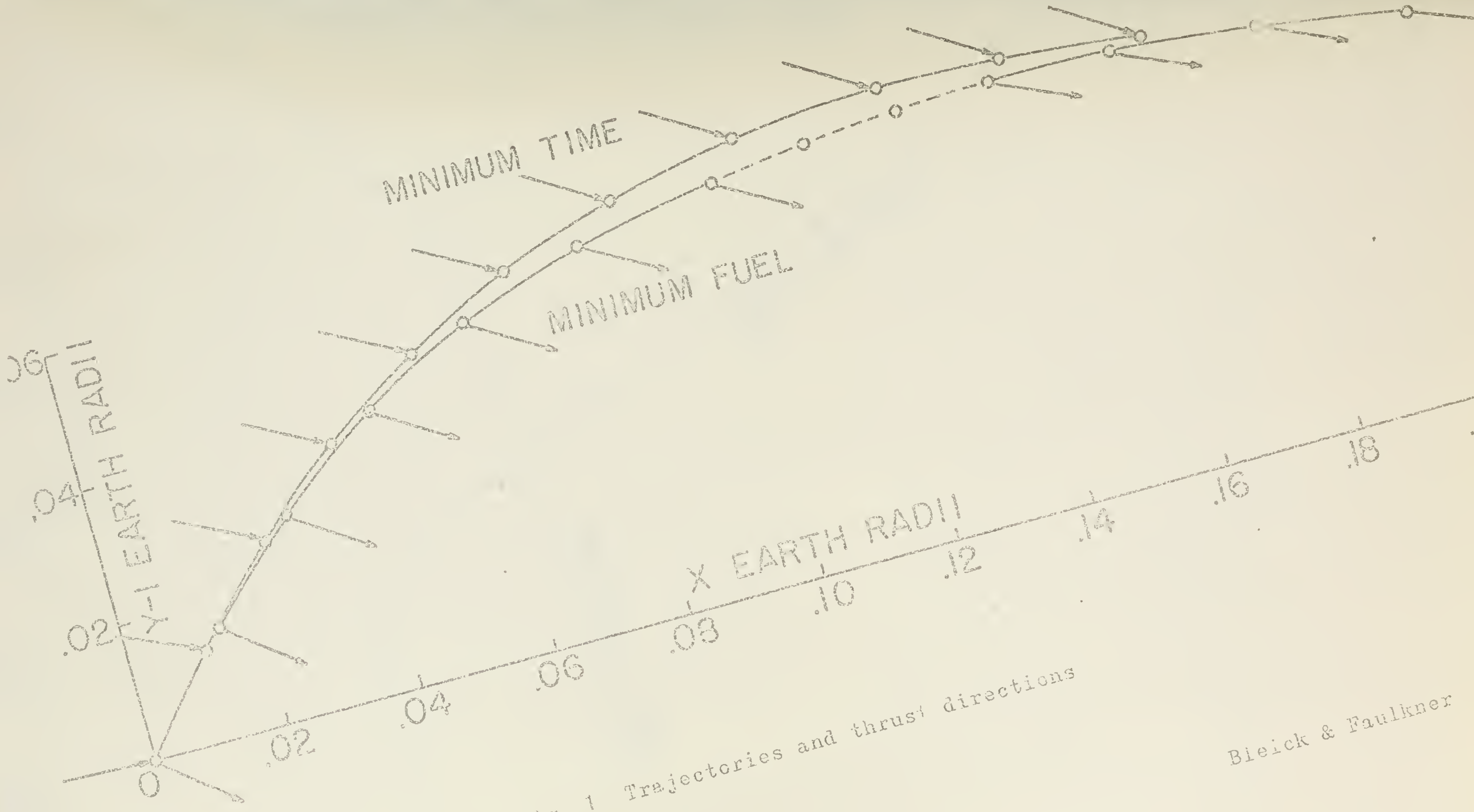

Fig. 1 Trajectories and thrust directions

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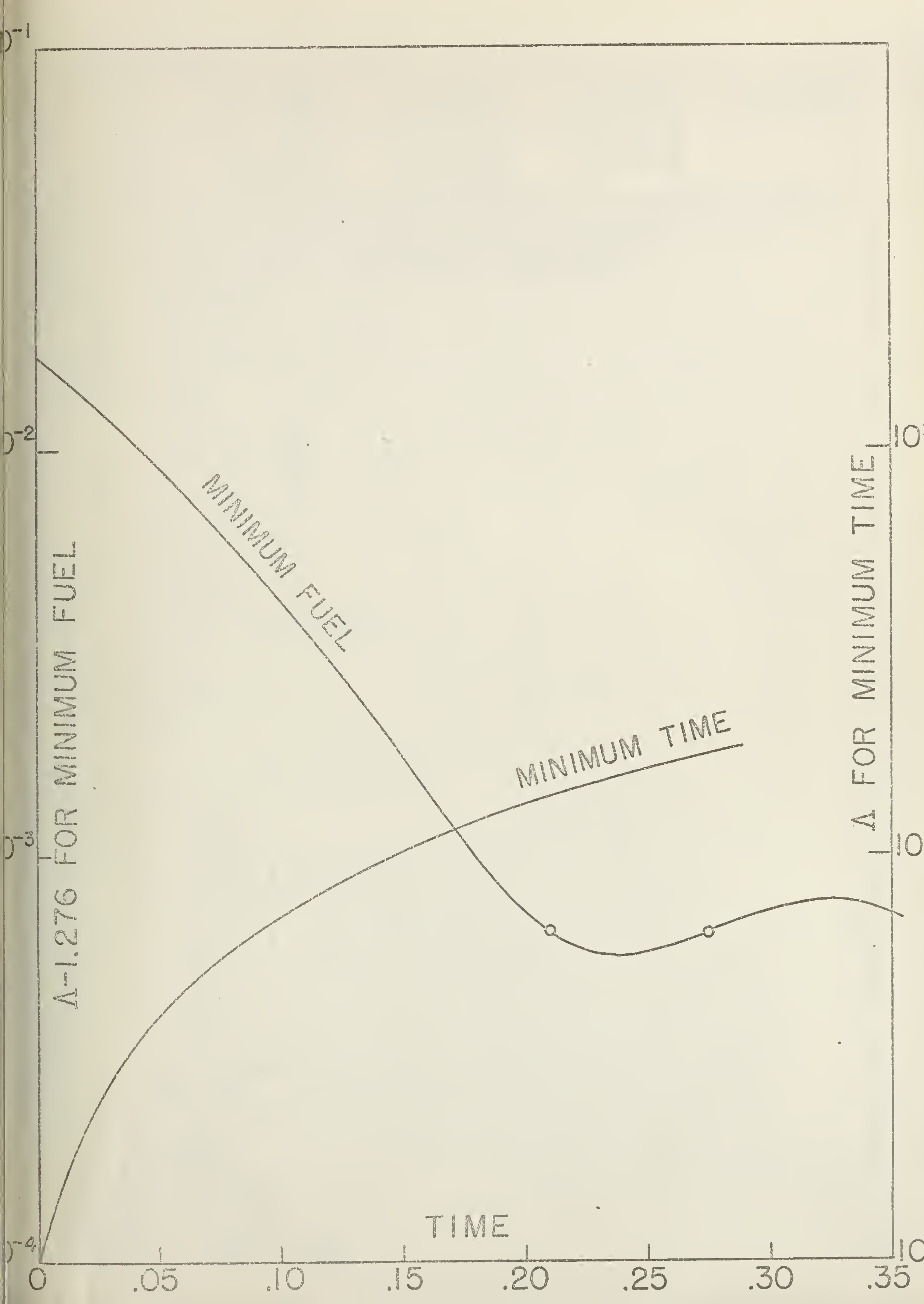

MINIMUM FUEL
MINIMUM TIME
Λ-1.276 FOR MINIMUM FUEL
Λ FOR MINIMUM TIME
TIME
10^2
10^1
10^0
0
.05
.10
.15
.20
.25
.30
.35

```
..JOB*BLEICK(33) - 3 MINUTE EXPRESS  7/31/63
  1 PROGRAM MINFUEL
CYVARS(1) =XU    YVRS(5)  =XLM1 YVRS(9)  =XLM2 YVRS(13)=XLM3 YVRS(17)=XLM4
C     (2) =XV         (6)  =XMU1      (10)=XMU2      (14)=XMU3      (18)=XMU4
C     (3) =XX         (7)  =XPI1      (11)=XPI2      (15)=XPI3      (19)=XPI4
C     (4) =XY         (8)  =XRO1      (12)=XRO2      (16)=XRO3      (20)=XRO4
CYVARS(21)=AA12 YVRS(25)=AA23 YVRS(29)=AA44 YVRS(33)=C2ASQ
C     (22)=AA13       (26)=AA24       (30)=A2LAM     (34)=C3ASQ
C     (23)=AA14       (27)=AA33       (31)=A3LAM     (35)=C4ASC
C     (24)=AA22       (28)=AA34       (32)=C1ASQ
  2 DIMENSION YVARS(35),AK(4,35),DY(35),YC(35),C(4),XU(500),XV(500),
   +B(6),XX(500),XY(500),TAU(500),CAPLAM(500),CGALAM(500),A2LAM(500),
   +P(500),CC(4),A(6,6),AI(6,6),DEL(6),CAPV(4),CAPVD(4),CIT1(4),
   +CIT2(4),CCIA2T2(4),AAA(4),AA(4,4),TVAR(6),CCIA2T(4)
  3 EQUIVALENCE (T,TVAR(1)),(T1,TVAR(2)),(T2,TVAR(3)),
   1              (EL,TVAR(4)),(EM,TVAR(5)),(EN,TVAR(6))
  4 REARTH = 20 925 000.
  5 GACCEL = 32.086
  6  TUNIT = SQRTF(REARTH/GACCEL)
  7    CCC = 10 000.
  8 COVERG = CCC/(GACCEL * TUNIT)
  9  FMDOT = 0.0036
 10  OMEGA = FMDOT * TUNIT
 11 VSTART = 0.585
 12 THETA  = 0.928
 13      T = 0.353 966 649
 14     T1 = 0.210 274 520
 15     T2 = 0.275 321 127
 16     EL =-0.820 214 924
 17     EM =-0.708 762 302
 18     EN =-1.181 456 519
 19      R = 1.075 698 925
 20      V = SQRTF(1.0/R)
 21 VSQDR  = V*V/R
 22     BB = 0.189 335 935
 23   TFIN = 0.28972 53036
 24 XSTEP  = TFIN/116.
 25 XU(1)  = VSTART * COSF(THETA)
 26 XV(1)  = VSTART * SINF(THETA)
 27 XX(1)  = 0.0
 28 XY(1)  = 1.0
 29 TAU(1) = 0.0
 31 A2LAM(1)= 0.0
 32  C(1)  = 0.0
 33  C(2)  = 0.5
 34  C(3)  = 0.5
 35  C(4)  = 1.0
    KK = 0
 36 DO 271  L=1,3
 37 XVAR = 0.0
 38 YVARS(1)  = XU(1)
 39 YVARS(2)  = XV(1)
 40 YVARS(3)  = XX(1)
 41 YVARS(4)  = XY(1)
 42 CAPLAM(1)= SQRTF(1.0 + EL*EL)
425 P(1) = 57.2957 * ATANF(EL)
 43 XA = COVERG * OMEGA
 44 CGALAM(1) = COVERG * XA * CAPLAM(1)
 45 DO 46  I=6,35
 46 YVARS(I) = 0.0
 47 DO 48  I=5,20,5
 48 YVARS(I) = 1.0
 49 N1 = T1/XSTEP + 1.0
 50 XN1 = N1
 51 STEP1 = T1/XN1
 52 N2 = (T2-T1)/XSTEP + 1.0
 53 XN2 = N2
 54 STEP2 = (T2-T1)/XN2
 55     N2 = N1 + N2
 56 N3 = (T-T2)/XSTEP + 1.0
 57 XN3 = N3
 58 STEP3 = (T-T2)/XN3
 59 N3 = N2 + N3
 60 SINB = SINF(BB)
 61 COSB = COSF(BB)
 62 CAPV(1) = V * COSB
 63 CAPV(2) = -V * SINB
 64 CAPV(3) = R * SINB
 65 CAPV(4) = R * COSB
 66 CAPVD(1) = -VSQDR * SINB
 67 CAPVD(2) = -VSQDR * COSB
```

```
68 CAPVD(3) = CAPV(1)
69 CAPVD(4) = CAPV(2)
70 N4 = N3 + 1
71 DO 206  K=2,N4
72 IF (N1+1-K)  75,73,73
73 STEP = STEP1
74 GO TO 79
75 IF (N2+1-K)  78,76,76
76 STEP = STEP2
77 GO TO 79
78 STEP = STEP3
79 DO 124  I=1,4
80 XC = XVAR + C(I) * STEP
81 DO 82   J=1,35
82 YC(J) = YVARS(J) + C(I) * AK(I-1,J)
83 XLAM = YC(5) + EL*YC(9) + EM*YC(13) + EN*YC(17)
84 XMU  = YC(6) + EL*YC(10) + EM*YC(14) + EN*YC(18)
85 CLAM = SQRTF(XLAM**2 + XMU**2)
86 COSP = XLAM/CLAM
87 SINP = XMU/CLAM
88 IF (N1+1-K)  91,89,89
89 XA = COVERG * OMEGA/(1.0 - OMEGA * XC)
90 GO TO 95
91 IF (N2+1-K)  94,92,92
92 XA = 0.0
93 GO TO 95
94 XA = COVERG*OMEGA/(1.0-OMEGA*(XC-T2+T1))
95 XR = SQRTF(YC(3)**2 + YC(4)**2)
96 DY(1) = -YC(3)/XR**3 + XA*COSP
97 DY(2) = -YC(4)/XR**3 + XA*SINP
98 DY(3) = YC(1)
99 DY(4) = YC(2)
100 G1X = (2.*YC(3)**2 - YC(4)**2)/XR**5
101 G1Y = 3.*YC(3)*YC(4)/XR**5
102 G2X = G1Y
103 G2Y = (2.*YC(4)**2 - YC(3)**2)/XR**5
104 DO 108  M=5,17,4
105 DY(M) = -YC(M+2)
106 DY(M+1) = -YC(M+3)
107 DY(M+2) = -G1X*YC(M) - G2X*YC(M+1)
108 DY(M+3) = -G1Y*YC(M) - G2Y*YC(M+1)
109 DO 110  M=1,4
110 AAA(M) = COSP*YC(4*M+2) - SINP*YC(4*M+1)
111 DO 112  M=1,3
112 DY(20+M) = XA*AAA(1)*AAA(M+1)/CLAM
113 DO 114  M=1,3
114 DY(23+M) = XA*AAA(2)*AAA(M+1)/CLAM
115 DY(27) = XA*AAA(3)*AAA(3)/CLAM
116 DY(28) = XA*AAA(3)*AAA(4)/CLAM
117 DY(29) = XA*AAA(4)*AAA(4)/CLAM
118 DY(30) = XA*XA*CLAM
119 DY(31) = XA*DY(30)
120 DO 122  M=1,4
121 CC(M) = YC(4*M+1)*COSP + YC(4*M+2)*SINP
122 DY(31+M) = CC(M)*XA*XA
123 DO 124  J=1,35
124 AK(I,J) = STEP*DY(J)
125 DO 126  J=1,35
126 YVARS(J) = YVARS(J) + ( AK(1,J)+2.*AK(2,J)+2.*AK(3,J)+AK(4,J) )/6.
127 XVAR = XVAR + STEP
128 TAU(K) = TAU(K-1) + STEP
129 XU(K)  = YVARS(1)
130 XV(K) = YVARS(2)
131 XX(K) = YVARS(3)
132 XY(K) = YVARS(4)
133 XLAM = YVARS(5) + EL*YVARS(9) + EM*YVARS(13) + EN*YVARS(17)
134 XMU  = YVARS(6) + EL*YVARS(10) + EM*YVARS(14) + EN*YVARS(18)
135 CLAM = SQRTF(XLAM**2 + XMU**2)
136 CAPLAM(K) = CLAM
137 COSP = XLAM/CLAM
138 SINP = XMU/CLAM
139 DO 140  M=1,4
140 CC(M) = YVARS(4*M+1)*COSP + YVARS(4*M+2)*SINP
146 DO 148  M=5,17,4
147 DY(M) = -YVARS(M+2)
148 DY(M+1) = -YVARS(M+3)
149 CGALAM(K) = COVERG*XA*CLAM
150 A2LAM(K) = YVARS(30)
151 IF (XLAM)  159,152,159
152 IF (XMU)   157,155,153
153 P(K) = 90.0
```

```
154 GO TO 165
155 P(K) = 0.0
156 GO TO 165
157 P(K) = -90.0
158 GO TO 165
159 P(K) = 57.2957 * ATANF(XMU/XLAM)
160 IF (XLAM)  161,165,165
161 IF (XMU)    164,162,162
162 P(K) = P(K) + 180.0
163 GO TO 165
164 P(K) = P(K) - 180.0
165 XLAMDOT = DY(5) + EL*DY(9) + EM*DY(13) + EN*DY(17)
166 XMUDOT   = DY(6) + EL*DY(10) + EM*DY(14) + EN*DY(18)
167 IF (N1+1-K)  183,168,183
168 DO 169  M=1,4
169 CIT1(M) = CC(M)
170 A2LAMT1 = XA*XA*CLAM
171 CLAMDT1 = COSP*XLAMDOT + SINP*XMUDOT
172 CGALDT1 = COVERG*XA*CLAMDT1
173 CLAMT1  = CLAM
174 C2ACGT1 = COVERG*XA*CC(2)
175 C3ACGT1 = COVERG*XA*CC(3)
176 C4ACGT1 = COVERG*XA*CC(4)
177 AT1 = XA
178 CGALMT1 = COVERG*XA*CLAM
179 QA2LMT2 = YVARS(30)
180 CA3LMT2 = 2.0*YVARS(31)/COVERG
181 DO 182  M=1,4
182 QCIA2T2(M) = YVARS(31+M)
183 IF (N2+1-K)  190,184,190
184 DO 185  M=1,4
185 CIT2(M) = CC(M)
186 A2LAMT2 = AT1*AT1*CLAM
187 CGALAM(K) = COVERG*AT1*CLAM
188 CLAMDT2 = COSP*XLAMDOT + SINP*XMUDOT
189 CLAMT2 = CLAM
190 IF (N4-K)  206,191,206
191 CLAMDT = COSP*XLAMDOT + SINP*XMUDOT
192 CGALDT = COVERG*XA*CLAMDT
193 C2ACGT = COVERG*XA*CC(2)
194 C3ACGT = COVERG*XA*CC(3)
195 C4ACGT = COVERG*XA*CC(4)
196 CGALMT = COVERG*XA*CLAM
197 QA2LMT = YVARS(30)
198 CA3LMT = 2.0*YVARS(31)/COVERG
199 DO 200 M=1,4
200 QCIA2T(M) = YVARS(31+M)
201 XR = SQRTF(YVARS(3)**2 + YVARS(4)**2)
202 DY(1) = -YVARS(3)/XR**3 + XA*COSP
203 DY(2) = -YVARS(4)/XR**3 + XA*SINP
204 DY(3) =  YVARS(1)
205 DY(4) =  YVARS(2)
206 CONTINUE
207 PRINT 208
208 FORMAT(1H06X2HEL13X2HEM13X2HEN13X2HBB13X2HT113X2HT213X1HT)
209 PRINT 210, EL,EM,EN,BB,T1,T2,T
210 FORMAT (7F15.9)
211 PRINT 212
212 FORMAT (1H06X2HN113X2HN213X2HN313X1HU14X1HV14X1HX14X1HY)
213 PRINT 214, N1,N2,N3,XU(N4),XV(N4),XX(N4),XY(N4)
214 FORMAT (3I15,4F15.7)
215 PRINT 216
216 FORMAT (1H05X4HCAPU11X4HCAPV11X4HCAPX11X4HCAPY)
217 PRINT 218, (CAPV(M), M=1,4)
218 FORMAT (4F15.7)
219 DO 224  I=1,4
220 A(I,1) = 0.0
221 B(I) = 0.0
222 DO 224  J=1,4
223 A(I,1) = A(I,1) + YVARS(4*I+J)*(DY(J)-CAPVD(J))
224 B(I)   = B(I)   + YVARS(4*I+J)*(CAPV(J)-YVARS(J))
225 DO 227  I=1,4
226 A(I,2) = AT1*CIT1(I) + (QCIA2T(I)-QCIA2T2(I))/COVERG
227 A(I,3) =-AT1*CIT2(I) - (QCIA2T(I)-QCIA2T2(I))/COVERG
228 DO 230  J=2,4
229 AA(1,J)= YVARS(19+J)
230 AA(2,J)= YVARS(22+J)
231 AA(3,3)= YVARS(27)
232 AA(3,4)= YVARS(28)
233 AA(4,4)= YVARS(29)
234 DO 236  I=1,4
```

```
235 DO 236  J=1,I
236 AA(I,J) = AA(J,I)
237 DO 239  I=1,4
238 DO 239  J=1,3
239 A(I,J+3) = AA(I,J+1)
240 A(5,1) = -CGALDT
241 A(5,2) = A2LAMT1 + CGALDT1 + CA3LMT - CA3LMT2 - XA*XA*CLAM
242 A(5,3) =-A2LAMT2 -(CA3LMT - CA3LMT2) + XA*XA*CLAM
243 A(5,4) = QCIA2T(2) - QCIA2T2(2) + C2ACGT1 - C2ACGT
244 A(5,5) = QCIA2T(3) - QCIA2T2(3) + C3ACGT1 - C3ACGT
245 A(5,6) = QCIA2T(4) - QCIA2T2(4) + C4ACGT1 - C4ACGT
246   B(5) = CGALMT - CGALMT1 - QA2LMT + QA2LMT2
247 A(6,1) = 0.0
248 A(6,2) = CLAMDT1
249 A(6,3) =-CLAMDT2
250 DO 251  J=2,4
251 A(6,J+2) = CIT1(J) - CIT2(J)
252   B(6) = CLAMT2 - CLAMT1
272 DO 274  I=1,N4
273 CGALAM(I) = CGALAM(N4) - CGALAM(I)
274 A2LAM(I) = A2LAM(N4) - A2LAM(I)
    IF (KK)  320,320,300
300 KK = KK-1
    DET = A(5,5)*A(6,6)-A(5,6)*A(6,5)
    EM = EM + (B(5)*A(6,6)-B(6)*A(5,6))/DET
    EN = EN + (B(6)*A(5,5)-B(5)*A(6,5))/DET
    PRINT 301
301 FORMAT (1H04X3HTAU8X6HCAPLAM7X6HCGALAM8X5HA2LAM/)
    PRINT 302, (TAU(I),CAPLAM(I),CGALAM(I),A2LAM(I),  I=1,N4)
302 FORMAT (4F13.9)
    GO TO 37
320 DB1 = (XU(N4)-CAPV(1))/CAPV(2)
    DB2 =-(XV(N4)-CAPV(2))/CAPV(1)
    DB3 = (XX(N4)-CAPV(3))/CAPV(4)
    DB4 =-(XY(N4)-CAPV(4))/CAPV(3)
    BB  = BB + (DB1+DB2+DB3+DB4)/4.
    SINB = SINF(BB)
    COSB = COSF(BB)
    CAPV(1) = V*COSB
    CAPV(2) =-V*SINB
    CAPV(3) = R*SINB
    CAPV(4) = R*COSB
    CAPVD(1)=-VSQDR*SINB
    CAPVD(2)=-VSQDR*COSB
    CAPVD(3)= CAPV(1)
    CAPVD(4)= CAPV(2)
    PRINT 208
    PRINT 210,  EL,EM,EN,BB,T1,T2,T
321 PRINT 216
322 PRINT 218, (CAPV(M), M=1,4)
323 DO 328  I=1,4
324 A(I,1) = 0.0
325   B(I) = 0.0
326 DO 328  J=1,4
327 A(I,1) = A(I,1) + YVARS(4*I+J)*(DY(J)-CAPVD(J))
328  B(I) =    B(I) + YVARS(4*I+J)*(CAPV(J)-YVARS(J))
253 CALL GAUSS3 (6, 0.1E-09, A, AI, KER)
254 PRINT 255, KER
255 FORMAT (5HOKER=I1)
256 IF (KER-2)  258,257,258
257 STOP 257
258 DO 261  I=1,6
259 DEL(I) = 0.0
260 DO 261  J=1,6
261 DEL(I) = DEL(I) + AI(I,J)*B(J)
262 DO 263  I=1,6
263 TVAR(I) = TVAR(I) + DEL(I)
264 BB = BB + V*DEL(1)/R
265 PRINT 208
266 PRINT 210, EL,EM,EN,BB,T1,T2,T
267 IF (T2-T)  303,303,307
303 IF (T1-T)  304,304,307
304 IF (T2)  307,305,305
305 IF (T1)  307,306,306
306 IF (T1-T2)  271,271,307
307 GO TO 275
271 CONTINUE
275 PRINT 276
276 FORMAT(1H04X3HTAU10X2HXU11X2HXV11X2HXX11X2HXY9X6HCAPLAM7X6HCGALAM8
   1X5HA2LAM9X1HP/)
278 PRINT 279, (TAU(I),XU(I),XV(I),XX(I),XY(I),CAPLAM(I),CGALAM(I),
```

```
    1 A2LAM(I),P(I),   I=1,N4)
279 FORMAT (8F13.9, F13.2)
280 STOP 280
281 END
    SUBROUTINE GAUSS3 (N,EP,A,X,KER)
    DIMENSION  A(6,6), X(6,6)
    DO 1 I=1,N
    DO 1 J=1,N
  1 X(I,J)=0.0
    DO 2 K=1,N
  2 X(K,K)=1.0
 10 DO 34 L=1,N
    KP=0
    Z=0.0
    DO 12 K=L,N
    IF(Z-ABSF(A(K,L)))11,12,12
 11 Z=ABSF(A(K,L))
    KP=K
 12 CONTINUE
    IF(L-KP)13,20,20
 13 DO 14 J=L,N
    Z=A(L,J)
    A(L,J)=A(KP,J)
 14 A(KP,J)=Z
    DO 15 J=1,N
    Z=X(L,J)
    X(L,J)=X(KP,J)
 15 X(KP,J)=Z
 20 IF(ABSF(A(L,L))-EP)50,50,30
 30 IF(L-N)31,34,34
 31 LP1=L+1
    DO 36 K=LP1,N
    IF(A(K,L))32,36,32
 32 RATIO=A(K,L)/A(L,L)
    DO 33 J=LP1,N
 33 A(K,J)=A(K,J)-RATIO*A(L,J)
    DO 35 J=1,N
 35 X(K,J)=X(K,J)-RATIO*X(L,J)
 36 CONTINUE
 34 CONTINUE
 40 DO 43 I=1,N
    II=N+1-I
    DO 43 J=1,N
    S=0.0
    IF(II-N)41,43,43
 41 IIP1=II+1
    DO 42 K=IIP1,N
 42 S=S+A(II,K)*X(K,J)
 43 X(II,J)=(X(II,J)-S)/A(II,II)
    KER=1
    RETURN
 50 KER=2
    END
    END
```